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Doppler Integration Intervals and Correlation [and Discussion]

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Doppler integration intervals and correlation

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Numerical tests have been carried out to determine the effect of changing the adopted integration time intervals used in the reduction of satellite Doppler data and to establish the contribution of geometrical correlation between adjacent Doppler counts. This has been done by using observational data obtained from a recent United Kingdom Doppler campaign involving 13 primary triangulation stations.

Nine different integration intervals were used, ranging from the smallest possible 4.6 s Doppler count to the largest practical interval of 2 min. The tests were carried out for both uncorrelated and correlated Doppler counts. The correlation model used was the standard geometrical correlation pattern for adjacent Doppler counts and range differences.

The results of the analysis seem largely to confirm the current mode of practice of using uncorrelated Doppler counts corresponding to time intervals ranging from 20 to 40 s.

1. Introduction

The problem of choosing the optimum integration interval for Doppler counts has been considered by several researchers (e.g. Kouba 1974; Hatch 1976). However, no systematic attempt seems to have been made to find the effect of different integration intervals on the parameters of the computation, such as the consistency of the solution, the efficiency of the data filtering and the computer processing requirements. Tests have been carried out at Nottingham to this end with a considerable amount of real Doppler data accumulated over a period of 3 months, by applying nine different Doppler integration intervals.

Another problem that has attracted much attention is that of the correlation existing between adjacent Doppler counts and ways of modelling it (see, for example, Brown 1970; Krakiwsky *et al.* 1972; Nesbø 1976). Two models have been tested at Nottingham, again by using this large amount of real data. It has been suggested that it would be impractical to correlate shorter integration intervals due to the large amount of computer time and storage required. An algorithm is given which overcomes this difficulty and which can be used to introduce correlation to any adopted integration interval, without any increase in computing requirements (Smith *et al.* 1976).

The bulk of the data was collected in the summer of 1976, during the first United Kingdom Doppler campaign (Ashkenazi *et al.* 1977*a*), with the use of a Canadian Marconi CMA-722B receiver. Altogether, 13 stations were occupied, located in five clusters of two or three stations each, throughout the United Kingdom. The test results are given in terms of the translation parameters from the broadcast ephemeris system to the OSGB 70 datum. However, one should point out that these values can only be regarded as a vehicle for the presentation of the results and not as reliable translation parameters. Since this analysis was done, several factors have emerged that alter the values of these parameters, but do not affect the conclusions of the paper. Additional tests were carried out by using an integration interval of 30 s with translocation data collected by JMR-1 receivers positioned on three triangulation pillars near Nottingham, separated from one another by 40–50 km.

The mechanics of applying different integration intervals, the theory for modelling the correlation, and the computer algorithm used are described in § 2. The results of the integration interval tests for the different uncorrelated and correlated models are given in § 3, followed by conclusions in § 4.

2. Theoretical and computational considerations

2.1. *Variable Doppler counts*

In most modern geodetic Doppler receivers the basic integration interval is approximately 4.6 s. However, these basic 4.6 s intervals are generally combined together during the data reduction stage to form any desired interval. UNDAP (University of Nottingham Doppler Adjustment Program) deals with this reduction and the subsequent solution as follows.

Table 1. Accumulation pattern of the basic (4.6 s) Doppler counts

nominal integration time/s	CMA-722B	JMR-1
4.6	25*1	26*1
10	3, 11*2	3, 10*2, 3
15	4, 7*3	4, 6*3, 4
20	5, 4, 4, 4, 4, 4	5, 4, 4, 4, 4, 5
24	5, 5, 5, 5, 5	5, 5, 5, 5, 6
30	6, 7, 6, 6	6, 7, 6, 7
40	9, 8, 8	9, 8, 9
60	13, 12	13, 13
120	25	26

The observed 4.6 s counts in each pass are first reduced to vacuum Doppler counts by applying the ionospheric (where necessary) and tropospheric corrections. These corrected counts are then accumulated into the appropriate intervals, as illustrated in table 1, and are used as uncorrelated observations in a two-dimensional least-squares adjustment and filtering procedure, followed by a three-dimensional adjustment to obtain a single pass solution. Finally, the normal equation matrices are combined and a cumulative solution is carried out (Ashkenazi *et al.* 1976).

2.2. *Basic correlation theory*

Methods of modelling the possible correlation between adjacent Doppler counts have been widely documented (Krakiwsky *et al.* 1972; Nesbø 1976). All modern geodetic Doppler receivers basically observe 'continuously integrated Dopplers (CIDs)' (Brown 1970) within a 2 min period which, because of their cumulative nature, may be considered uncorrelated. Most Doppler reduction programs use these data by subtracting successive cumulative counts from one another to obtain the basic interval counts, $\Delta N_{i,\,i+1}$. These can now be considered to be correlated and related to the uncorrelated CIDs, N_i, by the equation

$$\begin{bmatrix}\Delta N_{1,2}\\ \Delta N_{2,3}\\ \Delta N_{3,4}\end{bmatrix} = \begin{bmatrix}-1 & +1 & 0 & 0\\ 0 & -1 & +1 & 0\\ 0 & 0 & -1 & +1\end{bmatrix}\cdot\begin{bmatrix}N_1\\ N_2\\ N_3\\ N_4\end{bmatrix}, \tag{1}$$

or

$$\Delta N = R.N. \tag{2}$$

This relation leads to the covariance matrix for the observed Doppler vector, ΔN, in the form

$$\sigma_{\Delta N} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \tag{3}$$

and the corresponding weight matrix

$$W = \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}. \tag{4}$$

2.3. *Correlated Doppler models*

The type and degree of correlation between adjacent Doppler counts depends on the method used by the receiver for 'gating' these counts. With this in mind, two models have been developed and tested. In model A, it is assumed that full correlation exists between all adjacent Doppler counts within a pass. By contrast, in model B, one uses this same correlation procedure *within* each 2 min period, but not *between* two adjacent 2 min periods.

2.4. *Computational algorithm*

At first sight, the computation of the weight matrix for the Doppler counts (§ 2.2) appears to involve the inversion of a large square matrix. In the shortest integration interval (4.6 s), the resulting matrix would be so large that its inversion and storage would be considered impractical. However, because the covariance matrix has a regular form, the weight matrix for full correlation (model A) can be expressed as

$$W = \frac{1}{n+1}\begin{bmatrix} n & n-1 & n-2 & n-3 & \dots\dots & 1 \\ n-1 & 2(n-1) & 2(n-2) & 2(n-3) & \dots\dots & 2 \\ n-2 & 2(n-2) & 3(n-2) & 3(n-3) & \dots\dots & 3 \\ n-3 & 2(n-3) & 3(n-3) & 4(n-3) & \dots\dots & 4 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ 1 & 2 & 3 & 4 & \dots\dots & n \end{bmatrix}, \tag{5}$$

where n = the number of adjacent Doppler counts.

This matrix can be generated one element at a time. Lower triangular and leading diagonal elements are computed from

$$w_{ij} = (n+1-i)*j/(n+1), \tag{6}$$

whereas upper triangular and leading diagonal elements are obtained from

$$w_{ij} = (n+1-j)*i/(n+1),$$

where w_{ij} is the element of the weight matrix on the ith row and jth column. Clearly, it is unnecessary to store all, or even part, of the weight matrix in the computer, each element being generated as required.

The rejection of Doppler counts and the elimination of the corresponding rows in the matrix of (1) cause the covariance and the weight matrices to become block diagonal. This applies to the global (whole pass) weight matrix. Each separate block, however, will be a full matrix of the form given in (5). Similarly, the use of model B will cause the matrices to be block-diagonal, where each block represents an individual uncorrelated 2 min period which may, in turn, be block-diagonal, owing to the rejected observations.

3. Test results and discussion

3.1. *Uncorrelated Doppler counts*

The computed mean translation parameters, ΔX, ΔY, ΔZ, and their standard deviations, are listed in table 2, along with the mean number of passes accepted and the average amount of computer units used for the 13 stations and nine integration intervals.

Table 2. Mean translation parameters from broadcast to OSGB 70 for uncorrelated observations

Doppler interval/s	ΔX/m	ΔY/m	ΔZ/m	accepted passes	computing units
120	-371.2 ± 0.7	$+126.9 \pm 0.5$	-439.1 ± 0.5	22	41.3
60	-370.9 ± 0.8	$+126.5 \pm 0.6$	-439.3 ± 0.5	28	46.1
40	-371.3 ± 0.8	$+126.2 \pm 0.5$	-439.5 ± 0.5	30	51.6
30	-371.1 ± 0.8	$+126.3 \pm 0.6$	-439.7 ± 0.4	30	57.3
24	-371.4 ± 0.6	$+126.5 \pm 0.5$	-439.3 ± 0.5	31	63.8
20	-371.4 ± 0.6	$+126.3 \pm 0.5$	-439.3 ± 0.4	32	70.2
15	-371.5 ± 0.7	$+126.5 \pm 0.4$	-439.5 ± 0.4	32	83.8
10	-371.8 ± 0.7	$+126.4 \pm 0.5$	-439.4 ± 0.4	32	112.0
4.6	-371.5 ± 0.7	$+126.2 \pm 0.4$	-439.7 ± 0.5	33	211.9

Clearly, there is no significant difference in the corresponding values of the mean translation parameters and their standard deviations between the different integration intervals. However, as the integration interval drops below 20 s the computer costs begin to rise steeply (see figure 1). Furthermore, the use of longer intervals (greater than 30–40 s) leads to an unnecessary loss of data, both in terms of the total number of passes accepted (see figure 2) and also of the individual counts within accepted passes.

The choice of an optimum integration interval for a particular Doppler task depends not only on the required positional accuracy, but also on the resources available, e.g. manpower, equipment and facilities. The results suggest, that in general, one should use a Doppler integration interval somewhere between 20 and 40 s.

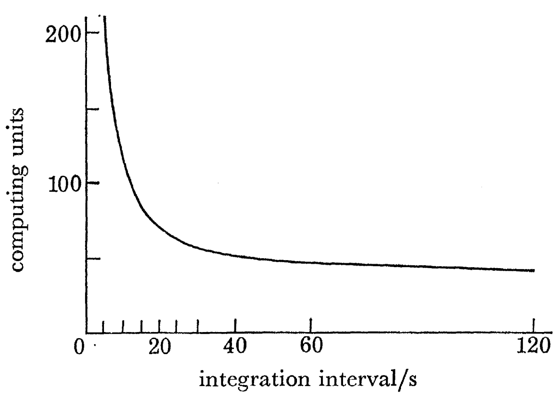

Figure 1. Mean computing units used plotted against length of integration interval.

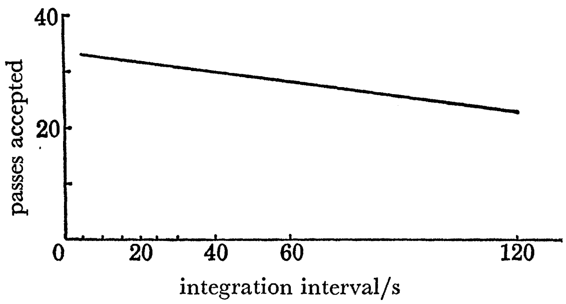

FIGURE 2. Mean number of passes accepted with varying integration interval.

3.2. *Correlated models A and B*

The results for the correlated models A and B are listed in tables 3 and 4 respectively.

The mean translation parameters in both models exhibit a larger random spread between different integration intervals than do the corresponding uncorrelated adjustment results especially in model A. Moreover, the standard deviations of the mean translation parameters

TABLE 3. MEAN TRANSLATION PARAMETERS FROM BROADCAST TO OSGB 70 FOR CORRELATION MODEL A

Doppler interval/s	ΔX/m	ΔY/m	ΔZ/m	accepted passes	computing units
120	-371.1 ± 0.7	$+126.5 \pm 0.6$	-439.2 ± 0.5	22	41.8
60	-370.5 ± 0.7	$+126.1 \pm 0.7$	-439.7 ± 0.5	28	46.8
40	-371.3 ± 1.0	$+125.2 \pm 0.6$	-439.4 ± 0.5	30	52.0
30	-371.1 ± 0.9	$+126.0 \pm 0.7$	-439.9 ± 0.5	31	58.0
24	-371.2 ± 0.9	$+126.4 \pm 1.1$	-439.2 ± 0.4	31	64.8
20	-372.0 ± 0.8	$+125.9 \pm 0.9$	-438.4 ± 0.4	31	70.9
15	-371.4 ± 1.0	$+126.5 \pm 0.9$	-439.7 ± 0.5	32	84.8
10	-372.5 ± 0.7	$+125.6 \pm 0.9$	-439.0 ± 0.6	33	113.5
4.6	-370.6 ± 1.2	$+124.7 \pm 0.6$	-439.7 ± 0.9	33	214.1

TABLE 4. MEAN TRANSLATION PARAMETERS FROM BROADCAST TO OSGB 70 FOR CORRELATION MODEL B

Doppler interval/s	ΔX/m	ΔY/m	ΔZ/m	accepted passes	computing units
120	-371.2 ± 0.7	$+126.9 \pm 0.5$	-439.1 ± 0.5	22	41.6
60	-371.1 ± 0.6	$+126.6 \pm 0.5$	-439.1 ± 0.5	28	46.7
40	-371.5 ± 0.8	$+126.7 \pm 0.6$	-439.5 ± 0.6	29	51.7
30	-371.2 ± 0.7	$+126.6 \pm 0.6$	-439.6 ± 0.6	30	57.5
24	-371.4 ± 0.7	$+126.8 \pm 0.6$	-439.5 ± 0.5	29	63.7
20	-371.6 ± 0.6	$+126.5 \pm 0.6$	-439.5 ± 0.5	30	70.7
15	-371.2 ± 0.7	$+126.7 \pm 0.6$	-440.2 ± 0.5	30	83.8
10	-371.6 ± 0.6	$+125.9 \pm 0.6$	-439.5 ± 0.6	31	111.7
4.6	-371.1 ± 1.0	$+124.8 \pm 0.4$	-440.3 ± 0.8	32	212.7

in model A are larger than both those for model B and the uncorrelated adjustment. However, there is no significant difference between the uncorrelated and the two correlated models in terms of the number of passes accepted and the overall computing cost.

The tests also confirmed the findings of other researchers (e.g. Kouba 1974; Krakiwsky *et al.* 1972) that the introduction of correlation does reduce the internal *a posteriori* standard errors of the calculated positions to a value that is independent of the integration interval. One should, however, remark that the internal consistency of the solution is a poor indication of the actual accuracy of the fix.

3.3. *Short range translocation data*

All the comparisons so far have been carried out by using translation parameters as criteria. However, as these are affected by the quality of the terrestrial network, it was decided to introduce some tests comparing chord distances. This was particularly suitable in this case because the Doppler observations were carried out in a simultaneous translocation mode, although they were subsequently processed by using a single point positioning program. Observational data from three points, ranging from 40 to 50 km apart, were available for this comparison. The results are listed in table 5 and are given in terms of the differences between the chord distances computed from the Doppler results on the one hand, and the terrestrial coordinates on the other.

TABLE 5. SHORT RANGE TRANSLOCATION: COMPARISON WITH TERRESTRIAL CHORD DISTANCES (METRES)

line... comparison	Harrowby – Charnwood (50 km)	Harrowby – Warren Hill (40 km)	Warren Hill – Charnwood (43 km)
terrestrial to uncorrelated Doppler	−2.27	0.47	0.01
terrestrial to model A Doppler	−4.21	−1.21	3.23
terrestrial to model B Doppler	−3.12	−0.32	0.95

Clearly, the correlated model A agrees poorly with the OSGB 70 chord distances. By contrast, both the uncorrelated and the (correlated) model B adjustments compare well with the terrestrial chord lengths, with the former being slightly better. However, considering that these series of observations were made with a very small sample (only three lines), the only clear conclusion that can be drawn is the weakness of model A.

4. CONCLUSIONS

(1) Given a large number of observational data, the accuracy of a position fix is unaffected by the adopted Doppler integration interval.

(2) However, under normal circumstances, a Doppler integration interval between 20 and 40 s is the most efficient.

(3) The best computational model tested here for reducing Doppler data appears to be the one involving no correlation between adjacent counts.

(4) Of the two correlated Doppler reduction models tested, the one with no correlation between adjacent 2 min periods was found to be superior to that with a continuous correlation.

(5) Should it be decided to adopt a correlated model, no additional computing costs will be incurred provided the algorithm proposed in this paper is used.

The work leading to this paper was completed in the Department of Civil Engineering of the University of Nottingham under the direction of Professor R. C. Coates.

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Discussion

P. G. Sluiter (*c/o Shell, EP/12, P.O. Box 162, The Hague, Netherlands*). The 0.9 m scatter in the results due to the use of different integration periods seems rather high. I noticed, however, that Mr McLintock rejected larger portions of passes for the longer integration periods. I should like to give as my opinion that the scatter in the results could be more due to not using identical pass lengths, rather than to the use of different integration intervals.

D. N. McLintock. This is certainly correct, but we were trying to illustrate the relative merits of using the different integration intervals, one aspect of which involves the amount of data accepted. The results quoted are those for an analysis with a real set of raw data and not for artificially controlled data.

A. L. Allan (*Department of Photogrammetry and Surveying, University College London, U.K.*). While it is fine to carry out analyses of results for correlations, I think that more analyses of instruments are needed to isolate the sources of error: the antenna, the frequency standard, etc. One way to help is to place various receivers at one point; at least this will remove orbital, ionospheric and tropospheric effects.

D. N. McLintock. Yes, this would certainly help to isolate the differences between the various receivers. However, we did not have any data available for this type of test.

J. C. Blankenburgh (*Continental Shelf Institute, Trondheim, Norway*). How much attention did Mr McLintock pay to the balancing of his data? What were the rejection criteria? There seems to be some difference between his rejection methods and the statements made by Professor Krakiwsky in the earlier paper.

D. N. McLintock. The data were balanced about the time of closest approach only when this point fell outside the central third of the pass. No balancing was carried out between passes. The rejection criteria applied were the normal high and low elevations for individual Doppler counts and the widely used elementary statistical checks on individual Doppler count residuals in a two-dimensional system. This is different from the method proposed by Professor Krakiwsky.

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